CIVIL ENGINEERING

Structural Analysis

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Structural Analysis

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1

Stability and Indeterminacy

1.1 INTRODUCTION

A structure is referred to as a system of connected parts used to support a load. Some of the examples related to Civil Engineering include buildings, bridges and towers etc. Various considerations such as safety, aesthetics and serviceability etc. of structure are taken into account while designing a structure. There are various unknown quantities in a structure such as axial forces, shear forces, bending moments, deflections and support reactions that are to be determined for purpose of analysis of a structure. These quantities are determined by using a number of independent equations which will be discussed in later part of book. In this chapter, we are going to study different types of beam, loads and supports. Stability of structure is also discussed in details. Determinacy of both type i.e., static as well as kinematic are also covered in detail in this chapter.

1.2 EQUATIONS OF STATIC EQUILIBRIUM

A structure that is initially at rest and remains at rest when subjected to a system of forces and couple is said to be in a state of static equilibrium. If a structure is in equilibrium, then all its members and parts are also in equilibrium.

(a) For 3-D

The necessary and sufficient conditions for a space structure are:

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$

$\Sigma M_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$

These 6 conditions of static equilibrium for 3-D structure implies that net force at any section in any member of structure and net moment of all the forces in any direction both are equal to zero. First 3 conditions resist the translational movement of structure while last 3 conditions resist rotational movement of structure.

NOTE On a space structure, if loads and reactions acting are concurrent, then conditions of static equilibrium are:

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$

(b) For 2-D

Although, all the structures are in 3-D in practical situations but as the principal load carrying elements of most structures are in 2-D stress condition, therefore, such elements are treated as 2-Dimensional structures also called as planar structures.

For a planar structure, conditions for static equilibrium are:

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$

The first two equations indicate that total force on structure in two perpendicular directions i.e., x and y axis and moment due to all the force about an axis perpendicular to x-y plane is zero.

NOTE

On a planar structure, if forces and reaction acting are concurrent, then conditions of static equilibrium are:

$\Sigma F_x = 0$ $\Sigma F_y = 0$

1.3 JOINTS

The various elements in a structure are connected to each other by joints and these are connected to foundations by special types of joints known as supports. Every connection whether it is joint or support is to serve two types of functions:

(a) **Kinematic function:** Each joint should ensure that elements which are connected by the joint should have identical displacement (translation and/or rotation) to provide the required integrity of structure.

(b) **Static function:** Each joint should transmit internal forces (axial forces, shear force, bending moment, twisting moment) from one connecting member to another.

1.3.1 Type of Joint

(a) **Rigid joint:** At a rigid joints, two (or more) members are so rigidly connected to each other such that there is no relative displacement (translation or rotation) of connecting members at the joint. This condition is required at a rigid joint to have complete transmissibility of internal forces from one member to another.

To have better understanding of rigid joint, consider a plane frame *ABC* as shown in figure (a) which is subjected to a horizontal load *P*.

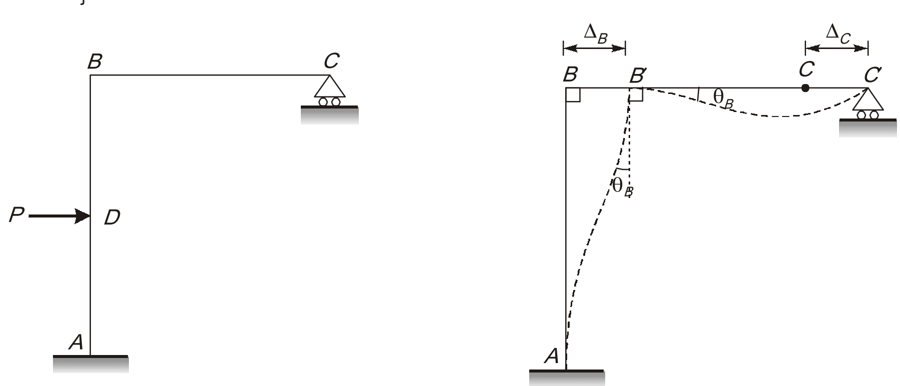


Fig. (a) Frame subjected to load

Fig. (b) Deflection of frame

Under the action of loading, displacement will occur in the frame as shown in figure (b).

Let the joint *B* rotate clockwise by θ_B due to load. As the joint *B* is rigid, the angle included between *AB* and *BC* will remain 90° as it was without the load. Hence, member *AB* and *BC* both will rotate by same angle θ_B.

> **NOTE** Rigid joint does not mean that there will be, no deformation in structure. Rigid joint can translate and rotate but it ensures compatibility of displacement of ends of connecting elements at joints.

(b) **Pinned joint:** As discussed in previous section, rigid joints resist rotation as well as translation, but pinned joint can resist translation only and is incapable of resisting rotation between the members.

Unlike rigid joint which can transmit bending moment from one member to another, pinned joint releases bending moment and hence bending moment is zero at pinned end. However, at pinned joint, transmission of axial and shear force is similar to that of rigid joint. In trusses, member are connected to each other by pinned joints.

To have a better understanding of pinned joint, consider a plane frame *ABC* with *B* as a pinned joint and subjected to a horizontal load *P* as shown in figure (a).

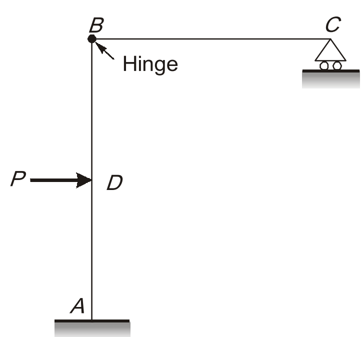


Fig. (a) Frame with pin joint subjected to horizontal load

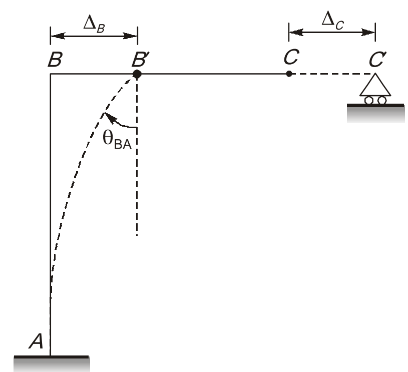


Fig. (b) Deflected shape of frame

Under the action of load, displacement will occur in frame as shown in figure (b).

Now, as the joint *B* is pinned , therefore relative angle between *AB* and *BC* will not remain 90° as it was before without load. It is to be noted that member *BC* won't have any force in it hence *AB* will behave as a cantilever. Portion between *B* and *C* will remain straight.

> **NOTE** Pinned joint when encountered in a beam is known as internal hinge (except at support). Consider a propped cantilever beam of length *L* in which an internal hinge is provided at centre of span subjected to load as shown in figure (a)

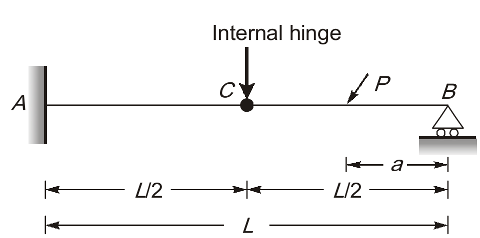


Fig. Internal hinge in beam

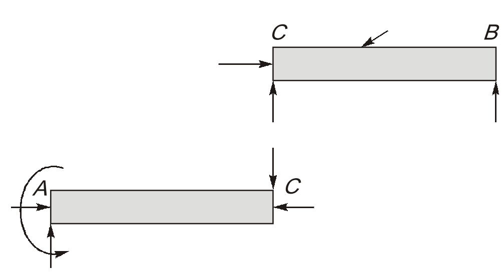


Fig. FBD of AC and CB

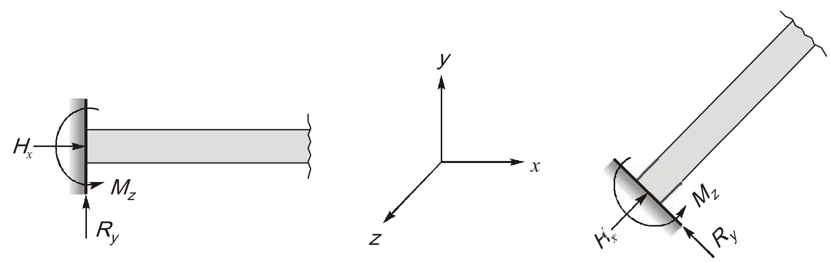
At hinge, *C*, axial force and shear force will transmit from *BC* to *CA* but moment will not transmit. So bending moment at hinge is zero.

1.4 SUPPORT SYSTEM

Structural members are joined together by rigid connections or flexible connections. Whenever loads act on the structural member, reactions are developed at supports to prevent translation or rotation of structural member. Different type of support are as follows:

1.4.1 2-D External Supports

(i) Fixed Support



At 2-D fixed supports, there can be 3 reactions:

(i) One vertical reaction (R_y) which prevent translation of beam in vertical direction.

(ii) One horizontal reaction (H_x) which prevent translation of beam in horizontal direction.

(iii) One moment reaction (M_z) which prevent rotation of beam about z-axis at support.

(ii) Hinge Support

Hinge support is represented by symbol Δ.

Fig. (i) Number of reactions = 2

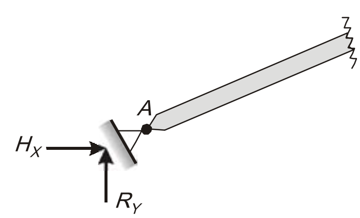


Fig. (ii) Number of reactions = 2

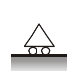
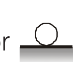
At hinge support, there can be 2 reactions:

(a) One vertical reaction R_y which prevent translation of beam in vertical direction.

(b) One horizontal reaction H_x which prevent translation of beam in horizontal direction.

Thus, hinge support can resist translational movement of beam but it cannot resist rotation of beam at support.

(iii) Roller Support

Roller support is represented by symbol  or .

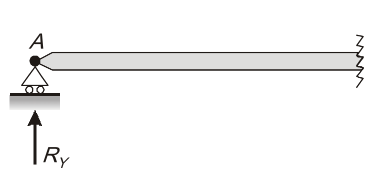


Fig. (i) Number of reactions = 1

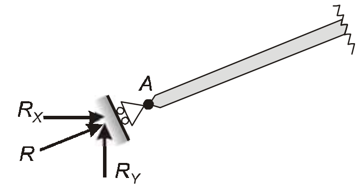


Fig. (ii) Number of reactions = 1

At roller support, there can be only one reaction that is a reaction that is perpendicular to contact surface. Thus, it can restrain translational movement of beam in one direction only.

(iv) Guided Roller Support

It differs from roller support in the fact that it can have two reactions.

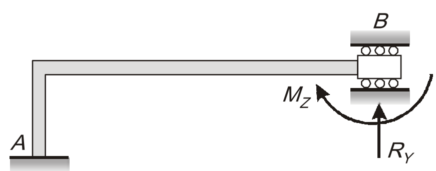


Fig. Number of reactions = 2

At guided roller support, there can be two reactions.

(i) One vertical reaction (R_y) which prevent translation of beam in vertical direction.

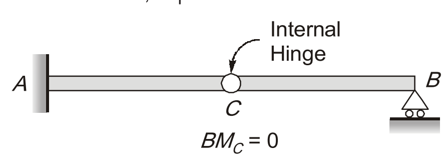
(ii) One movement reaction (M_z) which prevent rotation of beam about z-axis at support.

NOTE: In describing all the supports, it has been assumed that load acts in vertical direction only.

1.4.2 2-D Internal Joints

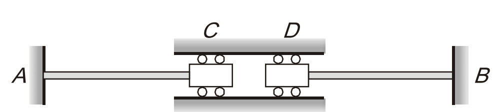
(a) Internal Hinge

At an inernal hinge, moment will be released so bending moment will be zero at that hinge from left side as well as right side. Therefore, it provides one additonal compatibility equation.

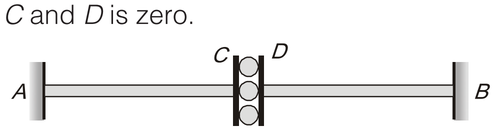


(b) Internal Roller

At internal roller, axial force or shear force can be released depending upon the position of rollers, so either axially force or shear force will be zero.



In fig. axially force at *C* and *D* is zero.



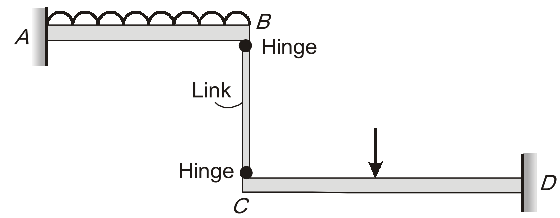
In fig. shear force at *C* and *D* will be zero i.e., $S_C = S_D = 0$

(c) Internal Link

If any member is connected by hinges at its end and subjected to no external loading in between then it can be termed as internal link and carry axial force only.

Here *BC* is a link, link *BC* carry only axial force

Also $BM_B = 0$ and $BM_C = 0$



NOTE: Internal link also provides additional equation for analysis of structure.

1.4.3 3-D Supports

(a) Fixed Support

At 3-D fixed support there can be six reactions:

(*i*) three reactions R_x, R_y and R_z

(*ii*) three moment reactions M_x, M_y and M_z

The fixed support is also called ***Built-in support***.

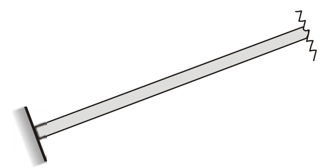


Fig. Number of reactions = 6

(b) 3-D Hinged Support

3-D hinged support there can be three reactions

(*i*) R_x (*ii*) R_y (*iii*) R_z

The 3-D hinged support is also called **'ball and socket joint'**.

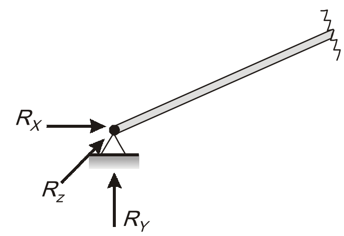


Fig. Number of reactions = 3

(c) Roller Support

At 3-D roller support there can be only one externally independent reaction which is perpendicular to the contact surface

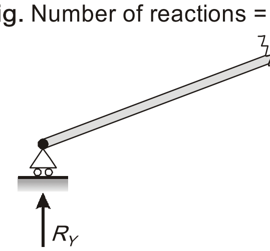


Fig. Number of reactions = 1

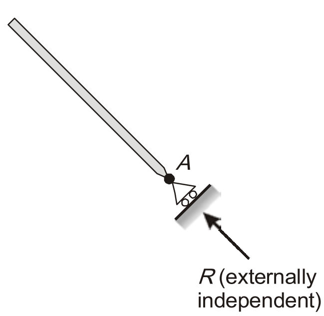


Fig. (i)

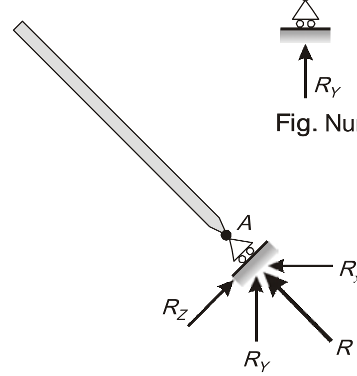


Fig. (ii)

In figure (ii), reactions at roller support *A*, R_x, R_y and R_z are externally dependent reactions which depends on reaction *R*.

1.5 STRUCTURE

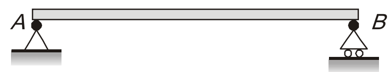
Structure is defined as a system of interconnected members that are assembled in a stable configuration and used to support loads under the equilibrium of external forces and internal reactions.

1.5.1 Elements of Structure

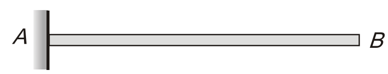
Some of the major elements of structure by which structures are fabricated are as follows:

(a) Beams: Beams are structural members which is predominantly subjected to bending. On the basic of support system beams can be classified as:

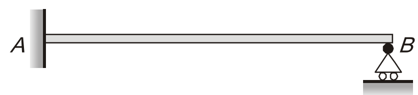
(*i*) Simply supported beam



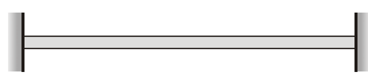
(*ii*) Cantilever beam



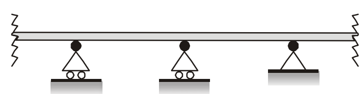
(*iii*) Propped cantilever



(*iv*) Fixed beam



(*v*) Continuous beam



(b) Columns: A column is a vertical compression member which is slender and straight. Generally columns are subjected to axial compression and bending moment as shown in figure.

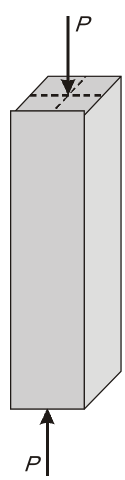


Fig. (i)

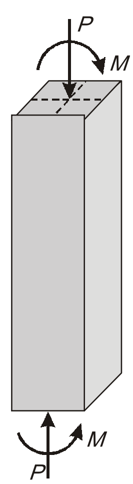


Fig. (ii)

(c) Tie Members: Tie members are tension members of trusses and frame, which are subjected to axial tensile force.

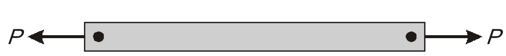


Fig. The rod

1.5.2 Types of Structures

(a) Trusses: A truss is constructed from pin jointed slender members, usually arranged in triangular manner. In trusses, loads are applied on joints due to which each member of truss subjected to only axial forces i.e., either axial compression or axial tension. Generally trusses are used when span of structure is large.

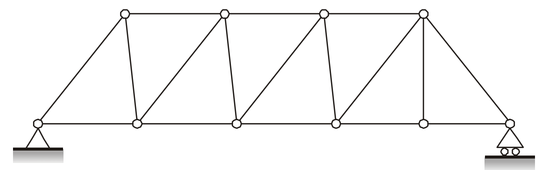


Fig. Truss

(b) Frames: A frame is constructed from either pin jointed or fixed jointed beam and columns. Generally loads are applied on beams and this loading causes axial force, shear force and bending to the members of frame.

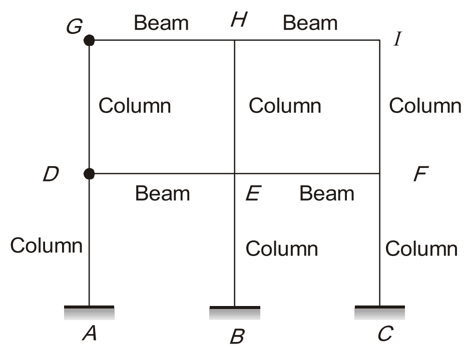


Fig. Frames

(c) Arches: Arches are used in bridges, dome roof, auditorium, where span of structures are relatively more due to external loading, Arch can be subjected to axial compression, shear force or bending moment.

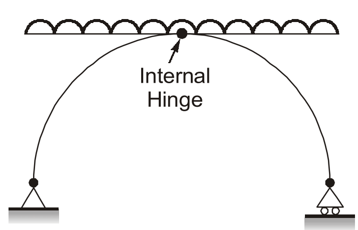


Fig. (i) Three Hinge Arch

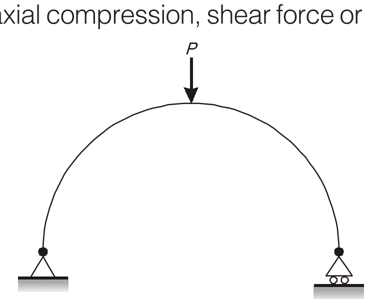


Fig. (ii) Two Hinge Arch

(d) Cables: Cables are used to support long span bridges. Cables are flexible members and due to external loading it is subjected to axial tension only.

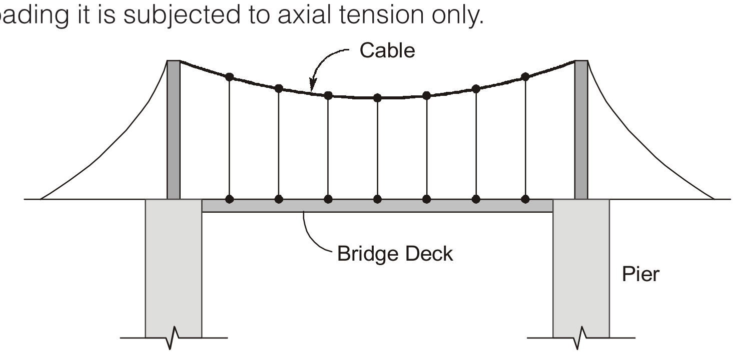
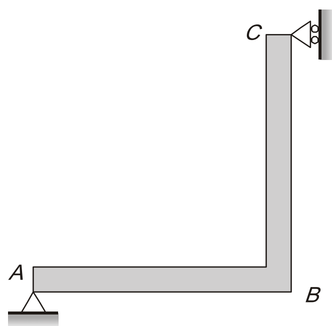


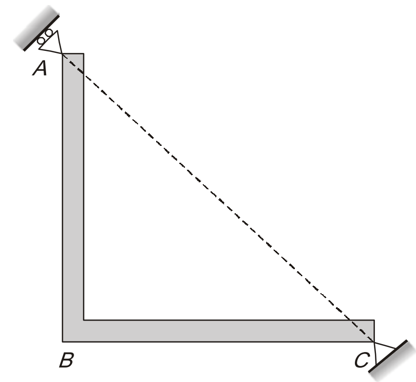
Fig. Cable and Bridge

Example 1.1 Which one of the following structures is stable?

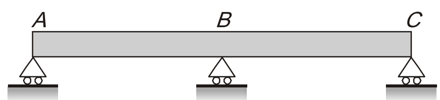
(a)



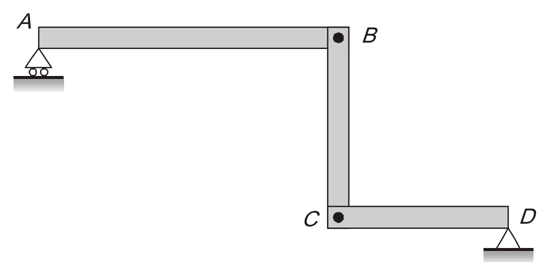
(b)



(c)

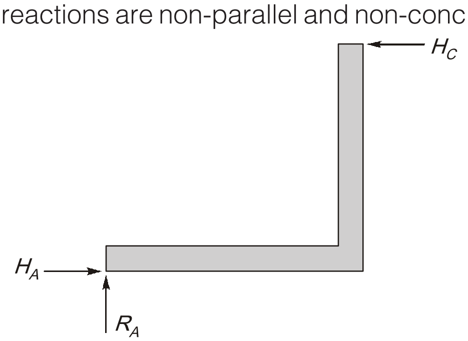


(d)

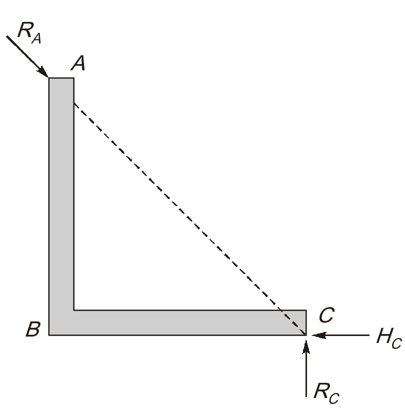


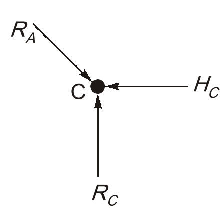
Ans. (a)

Member (a) is stable, since reactions are non-parallel and non-concurrent.

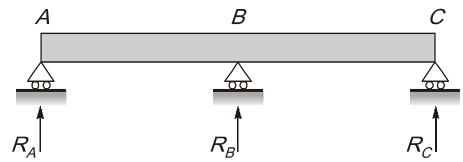


Member (b) is unstable since all the reactions are concurrent at *C*.

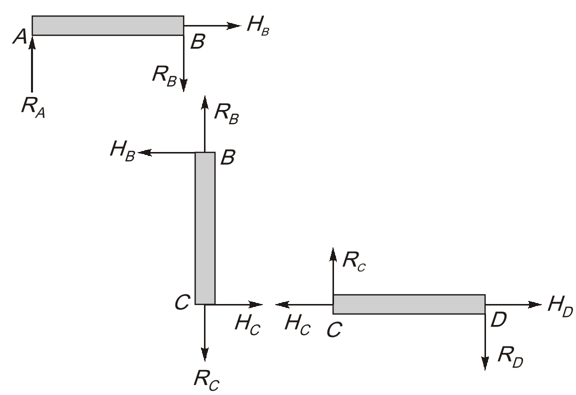




Beam (c) is unstable, since all three reactions are parallel.

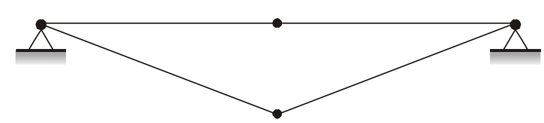


Structure (d) is unstable, since the member *AB* can move horizontally without any restrain. i.e. $\Sigma F_x \neq 0$



1.7.2 Internal Stability

For the internal stability, no part of the structure can move rigidly relative to the other part so that geometry of the structure is preserved, however small elastic deformations are permitted. To preserve geometry, enough number of members and their adequate arrangement is required. For the geometric stability, there should not be any condition of mechanism. Mechanism is formed when there are three collinear hinges, hence to preserve geometric stability there should not be three collinear hinges.



For 2-D truss the minimum number of members needed for geometric stability are:

$$m = 2j - 3$$

and for 3-D truss,

$$m = 3j - 6$$

where, j = Number of joint in truss

m = Number of Members required for geometrical stability.

All the members should be arranged in such a way that truss can be divided into triangular blocks. i.e. no rectangular or polygonal blocks.

Hence, for overall geometrical stability of truss:

(*i*) Minimum number of member should be present

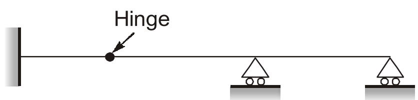
$m = 2j - 3$ (2-D truss)

and $m = 3j - 6$ (3-D truss)

(*ii*) There should be no condition of mechanism i.e. no three collinear hinges.

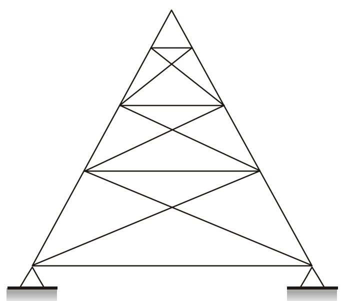
OBJECTIVE BRAIN TEASERS

Q.1 The degree of static indeterminacy of the beam given below is



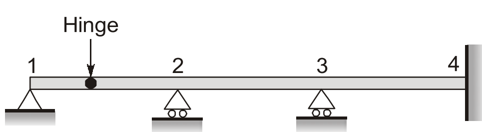
(a) zero (b) one
(c) two (d) three

Q.2 What is the total degree of static indeterminacy (both internal and external) of the triangular planar truss shown in the figure below?



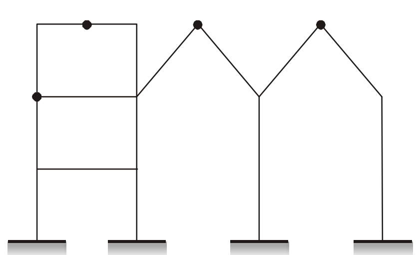
(a) 2 (b) 4
(c) 5 (d) 6

Q.3 The kinematic indeterminacy of the beam is



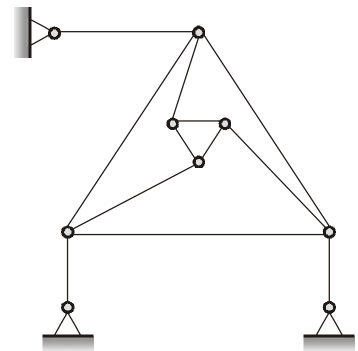
(a) 5 (b) 9
(c) 14 (d) 15

Q.4 For rigid frame shown in figure. Determine total degree of static indeterminacy.



(a) 10 (b) 11
(c) 13 (d) 8

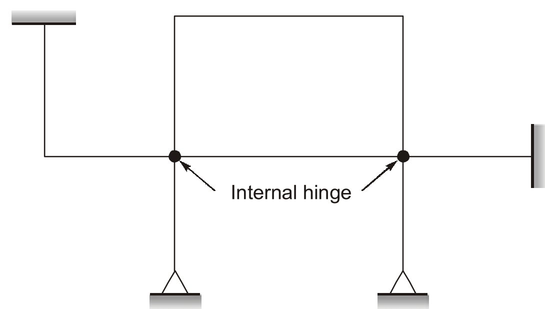
Q.5 The following two statements are made with reference to the plane truss shown below:



I. The truss is statically determinate
II. The truss is kinematically determinate

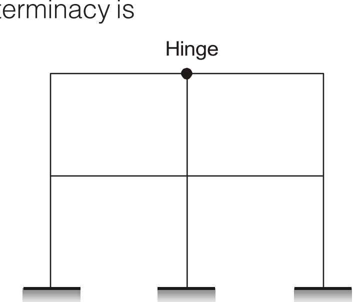
With reference to the above statements, which of the following applies?
(a) Both statements are true
(b) Both statements are false
(c) II is true but I is false
(d) I is true but II is false

Q.6 Find static indeterminacy of the Frame shown in figure



(a) 5 (b) 4
(c) 6 (d) 8

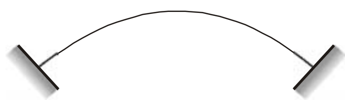
Q.7 The rigid Frame shown in figure, the statical indeterminacy is

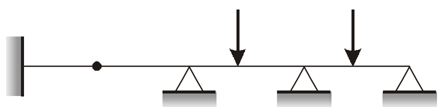


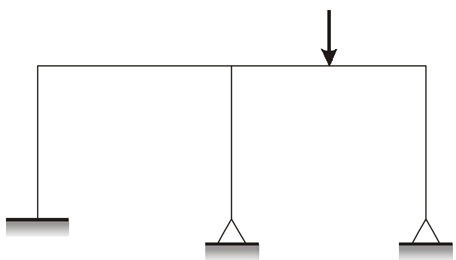
(a) 8 (b) 12
(c) 10 (d) 14

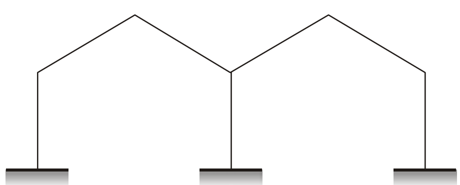
Q.29 Match **List-I** (Structural Frame) with **List-II** (Degree of static indeterminacy) and select the correct answer using the codes given below the lists:

List-I

A. 

B. 

C. 

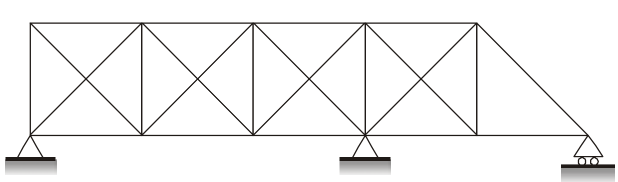
D. 

List-II
1. Five
2. Six
3. Two
4. Four

Codes:

	A	B	C	D
(a)	2	1	3	4
(b)	3	2	1	4
(c)	3	1	4	2
(d)	2	3	4	1

Q.30 The degree of static indeterminacy of the pin-jointed plane truss as shown in figure is



(a) 1 (b) 2
(c) 3 (d) 6

ANSWERS KEY

1. (b)	2. (b)	3. (b)	4. (a)	5. (d)
6. (b)	7. (c)	8. (c)	9. (a)	10. (d)
11. (a)	12. (d)	13. (c)	14. (a)	15. (d)
16. (b)	17. (a)	18. (c)	19. (d)	20. (b)
21. (a)	22. (8)	23. (b)	24. (b)	25. (c)
26. (0)	27. (12)	28. (a)	29. (c)	30. (d)

HINTS & EXPLANATIONS

1. (b)

$$D_S = r_e + 3m - r_r - 3(j + j')$$
$$r_e = 3 + 1 + 1 = 5$$

$m = 3$, $j = 3$, $j' = 1$

The hinge will create 2 members.

Number of internal reaction components released.

$$r_r = 1.0$$
$$\therefore \quad D_S = 5 + 9 - 1.0 - 3 \times (3 + 1) = 1.0$$

2. (b)

The total degree of indeterminacy is given by

$$D_S = m + r_e - 2j$$

Where,

m = number of members = 18

r_e = number of external reactions = 4

j = number of joints = 9

$$\therefore \quad D_S = 18 + 4 - 2 \times 9 = 4$$

3. (b)

The kinematic indeterminacy of the beam is

$$D_K = 3j - r_e + r_R$$

Given, $j = 5$

$r_e = 7$

$r_R = \Sigma(m' - 1) = \Sigma(2 - 1) = 1$

$$\therefore \quad D_K = 3 \times 5 - 7 + 1 = 9$$

$(\theta_1, \theta_{H1}, \theta_{H2}, \Delta_{Hx}, \Delta_{Hy}, \theta_2, \Delta_{2x}, \theta_3, \Delta_{3x})$

4. (a)

$m = 16, j = 15, r_e = 12$

$r_r = \Sigma(m' - 1)$
$= (3 - 1) + (2 - 1) + (2 - 1) + (2 - 1)$
$= 2 + 1 + 1 + 1 = 5$

First approach:

$D_{Se} = r_e - 3$
$= 12 - 3 = 9$

$D_{Si} = 3C - r_r$
$= 3 \times 2 - 5$
$= 1$

$D_S = D_{Se} + D_{Si}$
$= 9 + 1 = 10$

Second approach:

$D_S = 3m + r_e - 3j - r_r$
$= 3 \times 16 + 12 - 3 \times 15 - 5$
$= 48 + 12 - 45 - 5 = 10$

5. (d)

Degree of static indeterminacy

$D_s = m + r_e - 2j$

Here, $m = 12$, $j = 9$ and $r_e = 6$

$\therefore \quad D_s = 12 + 6 - 2 \times 9 = 0$

Degree of kinematic indeterminacy

$D_K = 2j - r_e - m$
$= 2 \times 9 - 6 - 0 = 12$

$\therefore$ Thus truss is statically determinate and kinematically indeterminate.

6. (b)

$D_S = D_{Se} + D_{Si}$

D_{Se} = external static indeterminacy

$D_{Se} = r_e - 3$

Here, $r_e = 3 + 2 + 2 + 3 = 10$

$\therefore \quad D_{Se} = 10 - 3 = 7$

and D_{Si} = Internal static indeterminacy

$D_{Si} = 3C - r_r$

Here, C = Number of closed loop = 1

r_r = internal reactions released
$= \Sigma(m' - 1) = (4 - 1) + (4 - 1) = 6$

$\therefore \quad D_{Si} = 3 \times 1 - 6 = -3$

Hence, $D_S = D_{Se} + D_{Si}$
$= 7 - 3 = 4$

Alternative approach:

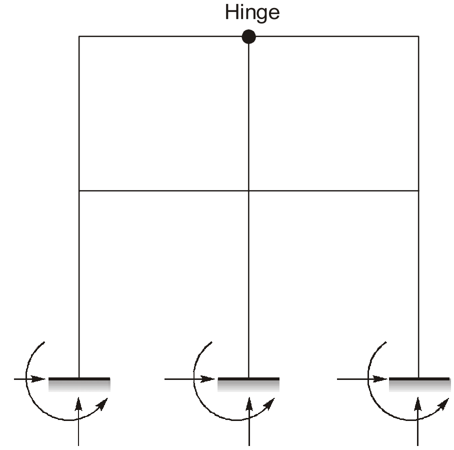
$D_S = 3m + r_e - 3j - r_r$

Here, $m = 9$, $j = 9$, $r_e = 10$ and $r_r = 6$

$\therefore \quad D_S = 3 \times 9 + 10 - 3 \times 9 - 6$

$D_S = 4$

7. (c)



First approach:

$D_S = D_{Se} + D_{Si}$

$D_{Se} = r_e - 3$

Here, $r_e = 9$

$\therefore \quad D_{Se} = 9 - 3 = 6$

and $D_{Si} = 3C - r_r$

Here, $C = 2$ and $r_r = \Sigma(m - 1) = (3 - 1) = 2$

$\therefore \quad D_{Si} = 3 \times 2 - 2 = 4$

Hence, $D_S = 6 + 4$

$D_S = 10$

Second approach:

$D_S = 3m + r_e - 3j - r_r$

Here, $m = 10$, $r_e = 9$, $j = 9$ and $r_r = 2$

$\therefore \quad D_S = 3 \times 10 + 9 - 3 \times 9 - 2$

$D_S = 10$

8. (c)

$D_S = 3m + r_e - 3j - r_r$

Here, $m = 12$, $j = 11$, $r_e = 3 + 3 + 3 + 3 = 12$

r_r = reaction released
$= \Sigma(m - 1) = (4 - 1) = 3$

$\therefore \quad D_S = 3 \times 12 + 12 - 3 \times 11 - 3$

$D_S = 12$

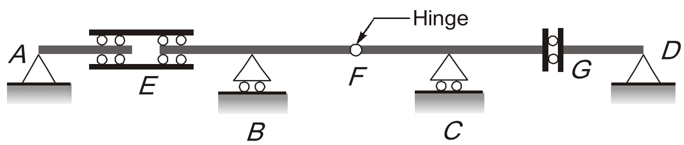
Alternative Approach:

$D_S = D_{Se} + D_{Si}$

$D_{Se} = r_e - 3$

CONVENTIONAL BRAIN TEASERS

Q.1 A continuous beam *ABCD* is shown below. The ratio of degree of kinematic indeterminacy when the members are extensile to the degree of kinematic indeterminacy when the members are inextensible is ________.



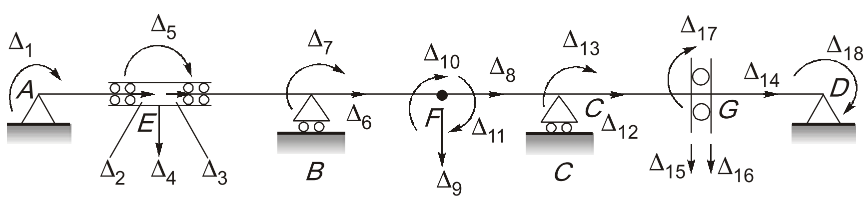
Solution:

$$j = 4,\ j' = 3,\ m = 6,\ r_e = 6,\ r_r = 3$$

Case-1: When members are extensible

$$D_K = 3\,(j + j') - r_e + r_r$$
$$D_K = 3\,(4 + 3) - 6 + 3 = 18$$

The 18 independent displacement components are identified below



Case-2: When members are inextensible

$$D'_K = 3\,(j + j') - r_e + r_r - m$$
$$D'_K = 3\,(4 + 3) - 6 + 3 - 6 = 12$$

The six horizontal displacements designated as Δ_2, Δ_3, Δ_6, Δ_8, Δ_{12} and Δ_{14} are prevented due to inextensibility of members.

$\therefore$ Required ratio $= \dfrac{D_K}{D'_K} = \dfrac{18}{12} = 1.5$

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